Dynamical response of Bose-Einstein condensates to oscillating gravitational fields

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joint work with Joel Lindkvist, Richard Howl and Ivette Fuentes

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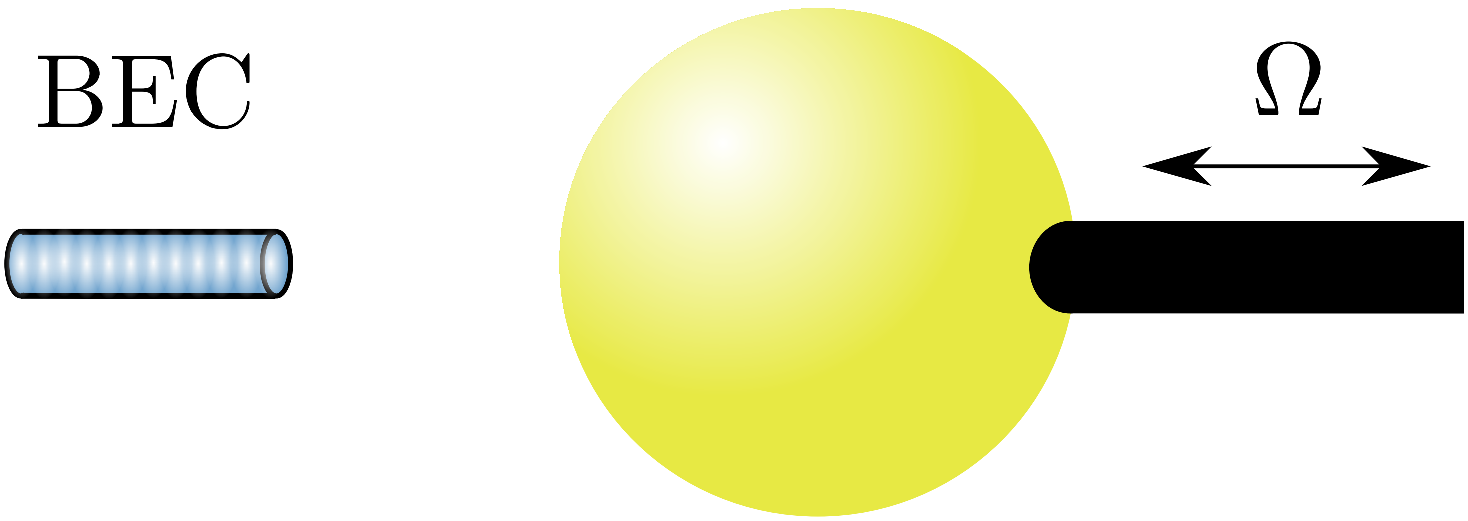
Setup



- BEC in a box trapping potential
- gold/tungsten sphere oscillates longitudinally
- gravitational field creates density perturbations

How can this become interesting?

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- BECs are very small → good for measuring gravitational fields of small objects
- may lead to precision measurements of Newton's constant and the inverse square law
- a step towards measurements of the gravitational field of masses in spatial superposition states

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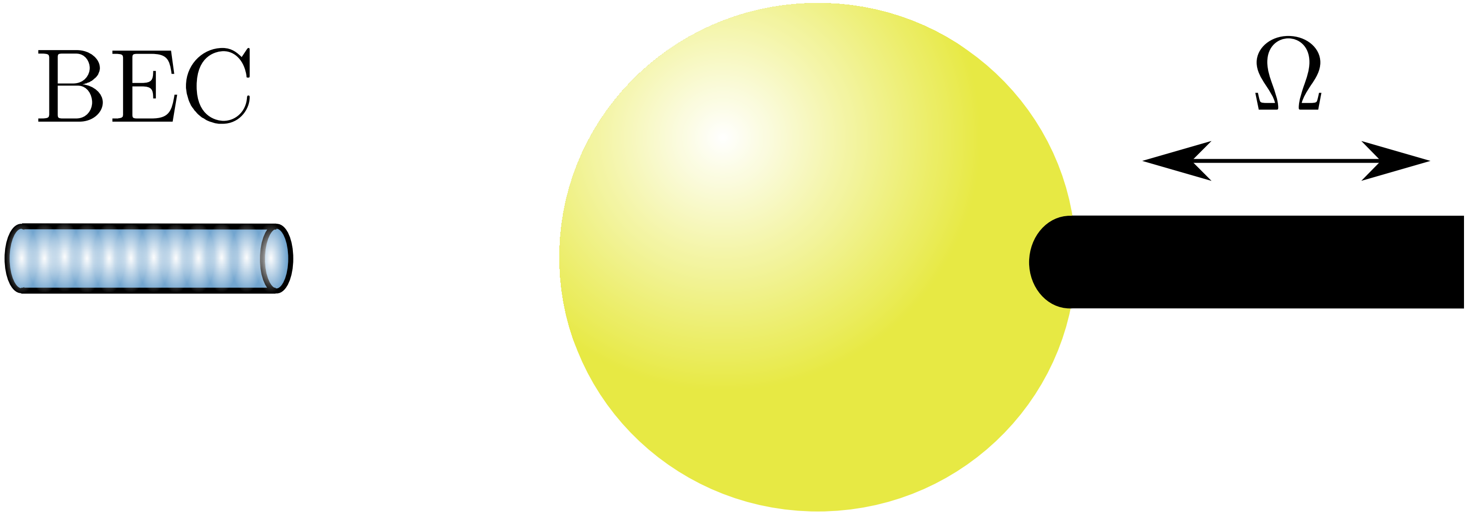
- gravity interacting with a quantum system
- a step beyond state of the art atom interferometry where stationary forces are measured
- it is another good reason to study phonons in a BEC, a quite complex, non-linear system

Back to our research



- source large in comparison to the BEC's width
  → effective 1d description

Gross-Pitaevskii equation

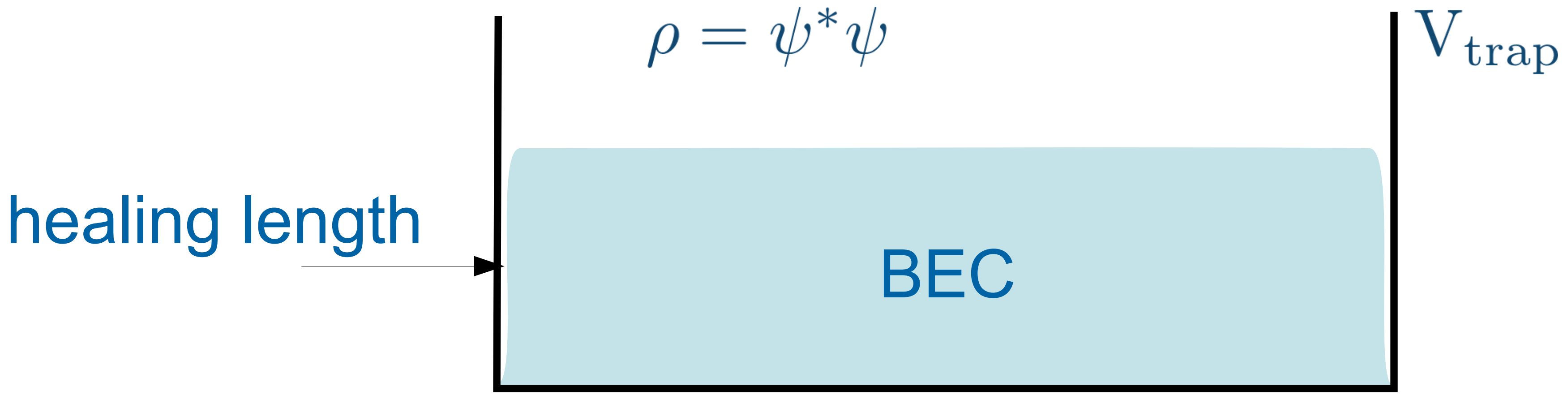
$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2}{2m}\partial_x^2 + V + \frac{\hbar^2}{2m}\lambda|\psi|^2\right]\psi$$

- non-linear Schrödinger equation for the wave function of the collective ground state ψ

Solution without gravity

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2}{2m}\partial_x^2 + V + \frac{\hbar^2}{2m}\lambda|\psi|^2\right]\psi$$

- numerical stationary solution for a unifrom trap potential is homogeneous up to healing length



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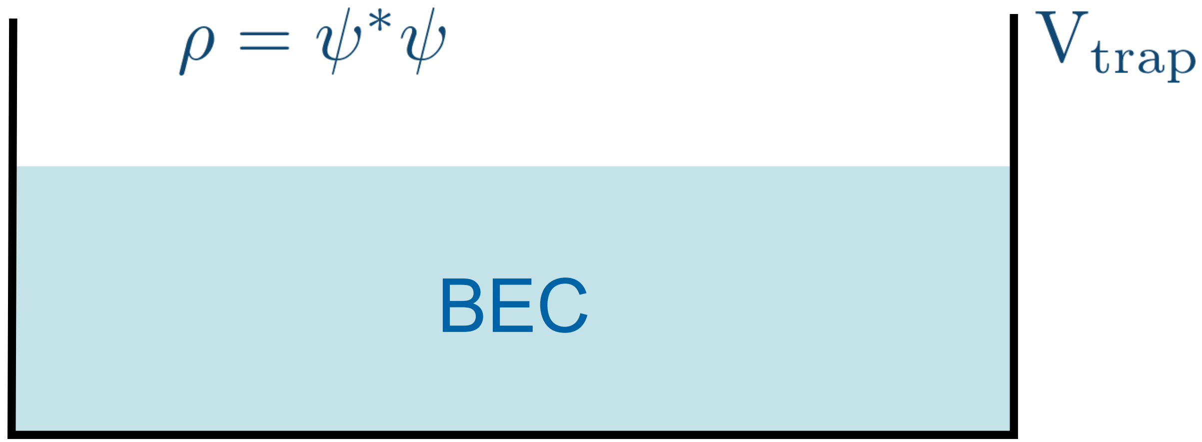
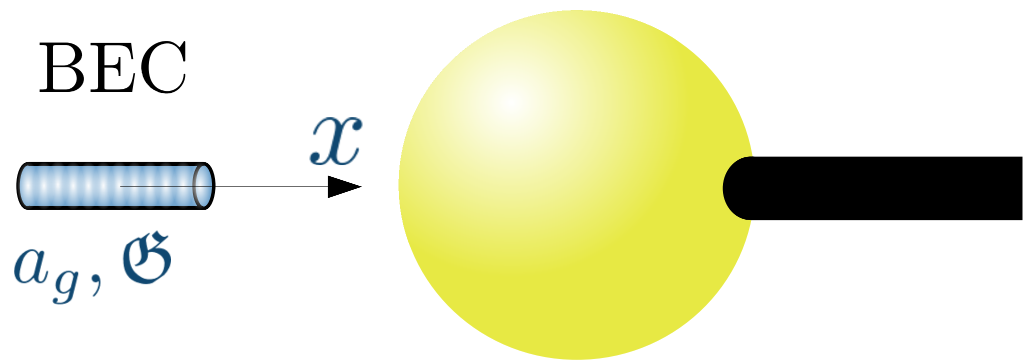


The gravitational field

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2}{2m}\partial_x^2 + V + \frac{\hbar^2}{2m}\lambda|\psi|^2\right]\psi$$

- external potential $\mathrm{V} = \mathrm{V}_{\mathrm{trap}} + \Phi$
- the gravitational potential $\Phi = \Phi_0 - a_g x - \mathfrak{G}x^2/2 + \ldots$ approximated at the center of the BEC



The gravitational field

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2}{2m}\partial_x^2 + V_{\text{trap}} + \Phi + \frac{\hbar^2}{2m}\lambda|\psi|^2\right]\psi$$

- gravitational field is a small perturbation of V
- perturbative solution $\psi = \sqrt{\rho_0}e^{i(\theta_0+\phi_0)}(1+\delta\psi)$
- where $\rho_0 = \psi_0^*\psi_0$ is the unperturbed density
- and $\mu = -\hbar\dot{\theta}_0$ is the unperturbed chemical potential

Bogoliubov-DeGennes equations

$$i\partial_t \delta\psi = -\frac{\hbar}{2m}\left(\frac{\rho_0'}{\rho_0}\partial_x + \partial_x^2\right)\delta\psi + \frac{\delta V}{\hbar}(1+\delta\psi) + \frac{\hbar}{2m}\lambda\rho_0\left(\delta\psi + \delta\psi^*\right)$$

- where $\rho_0 = \psi_0^* \psi_0$ is the unperturbed density
- density perturbation $\alpha = (\delta\psi + \delta\psi^*)/2$
- phase perturbation $\phi = -i(\delta\psi + \delta\psi^*)/2$
- Neumann boundary conditions: $\alpha' = \phi' = 0$

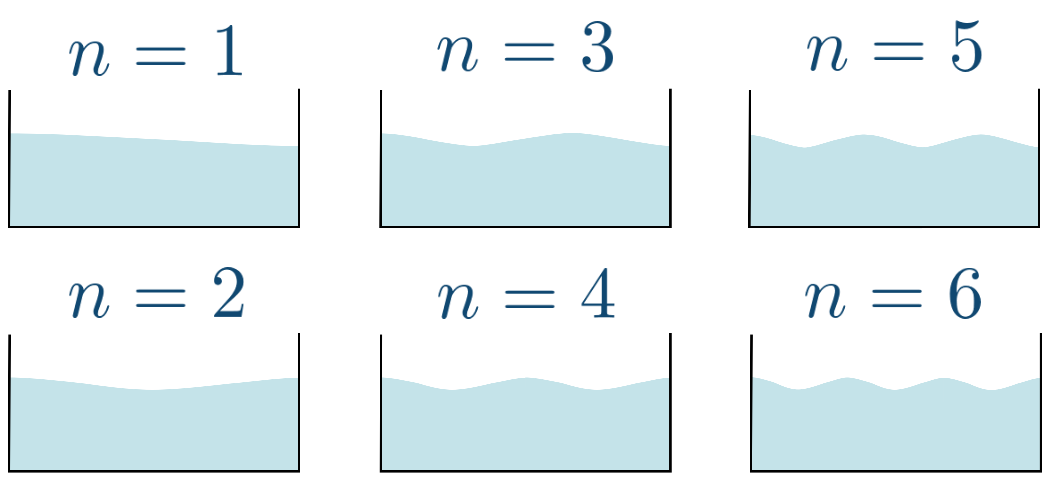
Mode Expansion

- mode expansion with Neumann boundary condition: $\alpha' = \phi' = 0$

$$\phi = \sum_{n\geq 1} g_n(t)\cos(k_n(x+L/2)) \qquad k_n = n\pi/L$$

anti-symmetric

symmetric



etc.

etc.

Mode Expansion

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$$\phi = \sum_{n\geq 1} g_n(t)\cos(k_n(x+L/2)) \qquad k_n = n\pi/L$$

$$\ddot{g}_n + \omega_n^2(1+S_n)g_n = D_n + \sum_{l\neq n} T_{nl}g_l$$

- only linear dispersion considered $\omega_n = c_0 k_n$
- number of phonons $\bar{N}_{n,\mathrm{c}} \propto \bar{g}_n^2$

Damping

$$\ddot{g}_n + \omega_n^2 (1 + S_n) g_n = D_n + \sum_{l \neq n} T_{nl} g_l$$

- phonon decay and phonon-phonon scattering due to non-linearities in the initial equations

Damping

$$\ddot{g}_n + \gamma_n \dot{g}_n + \omega_n^2 (1 + S_n) g_n = D_n + \sum_{l \neq n} T_{nl} g_l$$

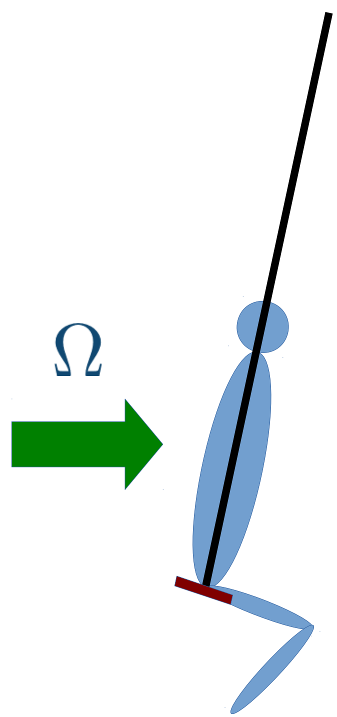
- phonon decay and phonon-phonon scattering due to non-linearities in the initial equations

  $\rightarrow$ incorporated into a single damping term

Direct driving

$$\ddot{g}_n + \gamma_n \dot{g}_n + \omega_n^2 g_n = D_n$$

$$D_n \quad \propto \quad (1-(-1)^n)\,\frac{\dot{a}_g}{L} - (1+(-1)^n)\,\frac{\dot{\mathfrak{G}}}{2}$$

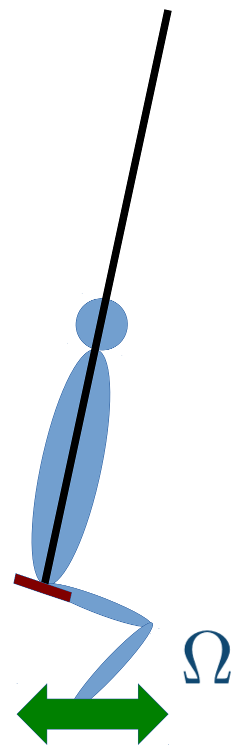


- resonance

$$\Omega = \omega_n$$

Parametric driving

$$\ddot{g}_n + \gamma_n \dot{g}_n + \omega_n^2 (1 + S_n) g_n = 0$$



$$S_n \quad \propto \quad \mathfrak{G}$$

- resonance

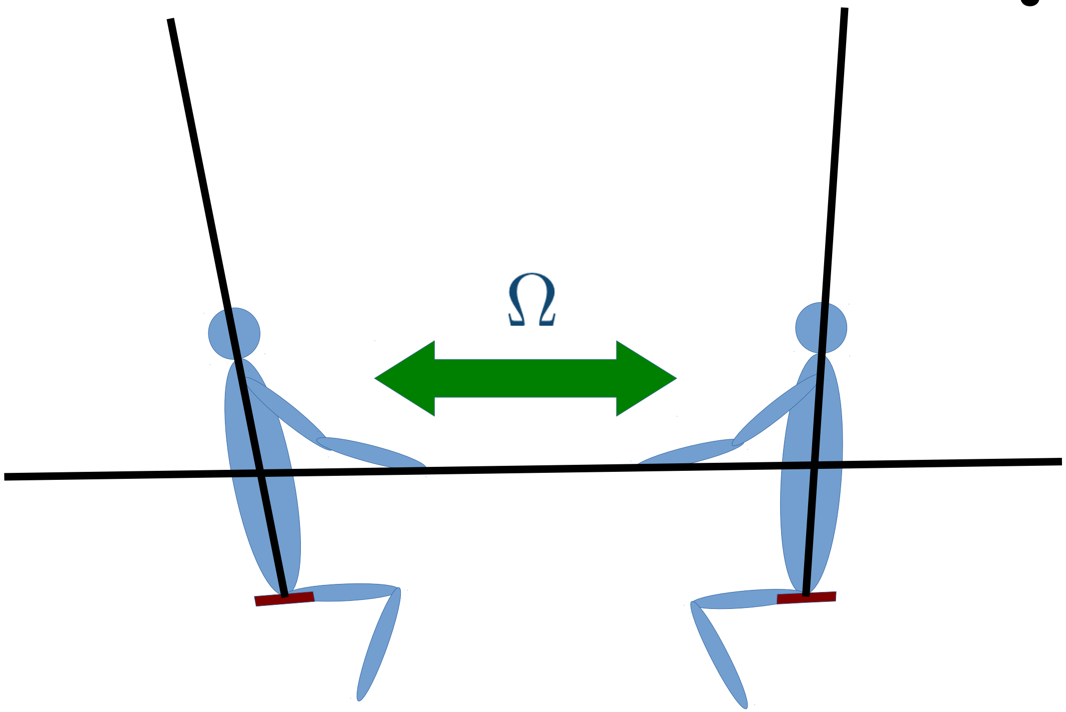
$$\Omega = 2\omega_n$$

- proportional to initial excitation

Mode coupling

$$\ddot{g}_n + \gamma_n \dot{g}_n + \omega_n^2 g_n = \sum_{l \neq n} T_{nl} g_l$$

$$T_{nl} \quad \propto \quad \left(1 - (-1)^{l+n}\right) \frac{a_g}{L} - \left(1 + (-1)^{l+n}\right) \frac{\mathfrak{G}}{2} \,,$$



- resonances

$$\Omega = |\omega_n - \omega_m|$$

$$\Omega = \omega_n + \omega_m$$

- proportional to initial excitation

Quantum field description

- atom field Hamiltonian

$$\hat{H} = \int_{\mathcal{V}} d^3x \, \hat{\Psi}^\dagger \left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \hat{\Psi} + \frac{g}{2} \int_{\mathcal{V}} d^3x \, \hat{\Psi}^\dagger \hat{\Psi}^\dagger \hat{\Psi} \hat{\Psi}$$

$$\hat{\Psi}(\vec{x}, t) = (\hat{\Psi}_0(\vec{x}) + \hat{\vartheta}(\vec{x}, t)) e^{-i\mu t/\hbar - i \int_0^t dt' \, \delta\mu(t')/\hbar}$$

- ground state atoms $\hat{\Psi}_0 = \hat{a}_0 \bar{\psi}_0$
- phonon field $\hat{\vartheta}(\vec{x}, t)$

Gravity effects on phonon field

- displacement
- one mode squeezing
- two mode squeezing and mode mixing

Effects on Quadratures

$$\sqrt{\rho_0}\alpha = \langle(\hat{\vartheta} + \hat{\vartheta}^\dagger)/2\rangle \qquad \sqrt{\rho_0}\phi = \langle -i(\hat{\vartheta} - \hat{\vartheta}^\dagger)/2\rangle$$

- displacement -> direct driving
- one mode squeezing

  -> parametric driving
- two mode squeezing and mode mixing
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  -> parametric driving
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- quantum metrology with different initial and final states and measurements can be applied

Quantum metrology

- Quantum Cramér-Rao bound $\sqrt{\Delta_\epsilon^{\mathrm{CR}}} \geq \frac{1}{\sqrt{\#_{\mathrm{rep}} I_\epsilon}}$
- 1000 initial squeezed phonons
- acceleration: $10^{-13}\,\mathrm{ms}^{-2}$ for $L = 200\,\mu\mathrm{m}$
- gradient: $10^{-10}\,\mathrm{s}^{-2}$ for $L = 500\,\mu\mathrm{m}$

  -> microgram scale measurable in principle

Experimental parameters for detection - Classical metrology

- measurement: phonon counting above thermal background with single phonon sensitivity
- measurement of acceleration with signal to noise ratio of the order 10 using direct driving/displacement
- assumptions: number of repetitions 10^4

$$\rho = 10^{13}\,\mathrm{cm}^{-3} \qquad L = 200\,\mu\mathrm{m}$$

$$T = 1\,\mathrm{nK} \qquad t_{\mathrm{exp}} = 10\,\mathrm{s}$$

atom species	M	$R_{\min}$	δ_R	a_Ω	$\Omega/2\pi$	N_a	L/ζ	d/L	$\bar{N}_{1,cr}$	$N_2^{\mathrm{lim},1}$	$\bar{N}_{1,\mathrm{th}}$
Rb-87	200 g	1 mm	2 mm	$2\times10^{-8}\,\mathrm{ms}^{-2}$	1.5 Hz	9×10^5	230	0.12	0.7	1.3	14
Yb-168	200 g	1 mm	2 mm	$2\times10^{-8}\,\mathrm{ms}^{-2}$	1.2 Hz	5×10^5	370	0.08	0.9	0.16	17
Rb-87	0.2 g	0.1 mm	0.2 mm	$2\times10^{-9}\,\mathrm{ms}^{-2}$	1.5 Hz	1×10^8	230	1.4	0.7	180	14
Yb-168	0.2 g	0.1 mm	0.2 mm	$2\times10^{-9}\,\mathrm{ms}^{-2}$	1.2 Hz	6×10^7	370	1	0.9	23	17

Experimental parameters for detection - Classical metrology

- measurement: phonon counting above thermal background with single phonon sensitivity
- measurement of acceleration with signal to noise ratio of the order 10 using mode coupling/squeezing
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atom species	M	R_{min}	δ_R	a_Ω	$\Omega/2\pi$	N_a^{min}	r_{12}	$N_{2,\mathrm{cr}}$	$\bar{N}_{2,0}$	$\bar{N}_{2,\mathrm{th}}$
Rb-87	200 g	1 mm	2 mm	$2\times10^{-8}\,\mathrm{ms}^{-2}$	4.4 Hz	5×10^4	0.3	0.4	5	4
Yb-168	200 g	1 mm	2 mm	$2\times10^{-8}\,\mathrm{ms}^{-2}$	3.7 Hz	1×10^4	0.8	0.4	1	4
Rb-87	0.2 g	0.1 mm	0.2 mm	$2\times10^{-9}\,\mathrm{ms}^{-2}$	4.4 Hz	7×10^6	0.02	0.4	700	4
Yb-168	0.2 g	0.1 mm	0.2 mm	$2\times10^{-9}\,\mathrm{ms}^{-2}$	3.7 Hz	1×10^6	0.07	0.4	100	4

Possible Measurement Processes

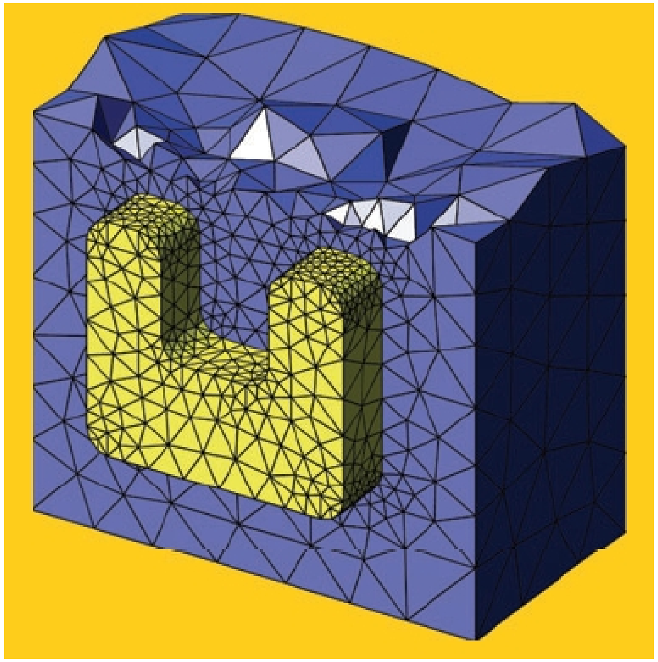
- resonant and non-resonant absorbtion imaging (Andrews et al. Science 273 84-87 (1996), Stamper-Kurn PRL 81, 500-503 (1998), Schley et al. PRL 111, 055301 (2013), Steinhauer Nature Phys. 10, 864 (2014))
- "heterodyne detection" (Katz et al. 93, 220403 (2004), Tozzo et al. condensates," PRA 69, 053606 (2004))
- time of flight measurements
- light-phonon coupling in cavity (optomechanics)
- coupling to quantum dots (Bruderer and Jaksch, NJP 8, 87 (2006))

Next Step: Numerical Description Marie-Curie IF - PhoQuS-G

- overcome limits of perturbative approach
- obtain full description of preparation and measurement processes
- consider other trap geometries

The Discontinuous Galerkin Time Domain (DGTD) Method

- finite element method used extensively for hydrodynamics simulations
- Madelung representation allows for hydrodynamic formulation of Gross Pitaevskii equation
- useful for constant trap geometries, in particular uniform trap
- already established hydrodynamic formulation of Maxwell's equations enables numerical implementation of light-phonon coupling -> BEC optomechanics



Busch et al., Laser Photonics Rev. 5, No. 6, 773–809 (2011)

Summary

- oscillating gravitational field induces displacement, single-mode squeezing, two-mode squeezing and mode mixing
- parametric processes more efficient than direct driving, but excited initial state necessary
- detection of gravitational field of 100 mg sphere with a BEC seems possible with state of the art technology
- application of quantum metrology to the phonon quantum field -> engineering possible
- next step: numerical implementation

Thank you very much for your attention

Dennis Rätzel, Joel Lindkvist, Richard Howl and Ivette Fuentes "Dynamical response of Bose-Einstein condensates to oscillating gravitational fields" NJP 20 (2018) 073044